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Quasi-linearization of non-linear systems under random vibration by probabilistic method †

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Abstract

Vibration of a non-linear system under random parametric excitations was evaluated by probabilistic methods. The non-linear characteristic terms of a system were quasi-linearized and excitation terms remained as they were. An analytical method where the square mean of error was minimized was used. An alternative method was an energy method where the damping energy and restoring energy of the linearized system were equalized to those of the original non-linear system. The numerical results were compared with those obtained by Monte Carlo simulation. A new method was proposed from the comparison of those results. Finally the results obtained by the combined method showed good agreement with those obtained by Monte Carlo simulation.

Keywords: Nonlinear system; Quasi-linearization; Probabilistic method; Monte Carlo simulation

1. Introduction

The importance of vibration evaluation for systems under fluctuating exciting forces has increased as it is continuously demanded that weight and cost for a structure be reduced and the performance be improved [1]. As irregularities are included in structural characteristics and excitation characteristics in many cases, it may be required to perform dynamic response evaluation and reliability analysis for nonlinear structure under random parametric excitations [2, 3].

The basic idea of statistical linearization is substituting a nonlinear structure with an equivalent linear system that has similar behaviors to the original system. One effective method to treat the problem is quasi-linearization, in which the nonlinear terms of a system are linearized and the parametric excitation terms remain unchanged. Different criteria have been used for the linearization procedure. A popular one is to minimize the mean-square value of the differences between the linearized system and the original system [4-6], while another is to equalize the expectation values for the dissipation energy and restoring energy of the two systems[7, 8]. In the present investigation, quasi- linearization methods with two different criteria are applied to systems with one degree of freedom and two degrees of freedom. Equations for the second- and fourth-order stationary moments are obtained by using Ito's stochastic differential rule. The equivalent linearization coefficients are calculated and the moments equations are solved by iteration.

Numerical results obtained by using two different linearization criteria are compared with those obtained by Monte Carlo simulation. The comparison shows that the results obtained by the two linearization criteria cannot show good agreement with those from the Monte Carlo simulation. Thus, a new method is proposed to combine the two linearization

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criteria. Numerical results obtained by the new combination method show good agreement with those from the Monte Carlo simulation.

2. Quasi-linearization of nonlinear systems

2.1 One DOF system

An equation of general nonlinear systems parametric excitations can be written as Eq. (1) [5, 9].

$$\ddot{Y}_j(t) + h_j\left(Y(t), \dot{Y}(t)\right) = \xi_j(t) + \sum_{i=1}^{n}\left[a_{ji}\dot{Y}(t)\eta_i(t) + b_{ji}Y_i(t)\gamma_i(t)\right] \quad (1)$$

In this section, we analyze a system with cubic nonlinear damping and stiffness as Eq. (2).

$$\ddot{Y} + 2\alpha_1\dot{Y} + \lambda\dot{Y}^3 + \Omega_1^2 Y + \delta Y^3 = \dot{Y}\eta(t) + Y\gamma(t) + \xi(t) \quad (2)$$

We assume that the spectral densities of Gaussian white noise $\eta(t)$, $\gamma(t)$, and $\xi(t)$ are $K_{\eta\eta}$, $K_{\gamma\gamma}$ and $K_{\xi\xi}$ respectively and three excitations are not coupled for convenience. A nonlinear system (2) can be substituted with an equivalent quasi-linear system as follows:

$$\ddot{Y} + 2\alpha\dot{Y} + \Omega^2 Y = \dot{Y}\eta(t) + Y\gamma(t) + \xi(t) \quad (3)$$

where α and Ω and are equivalent linearization coefficients. Let $X_1 = Y$, $X_2 = \dot{Y}$, then Ito's statistical equation [5] is derived from Eq. (3).

$$dX_1 = X_2 dt\,, \quad (4)$$

$$dX_2 = (-2\alpha X_2 + \pi K_{\eta\eta}X_2 - \Omega^2 X_1)dt + [2\pi(K_{\gamma\gamma}X_1^2 + K_{\eta\eta}X_2^2 + K_{\xi\xi})]^{\frac{1}{2}} dB(t)\,, \quad (5)$$

where $B(t)$ is a unit Wiener process and

$$E[dB(t_1)dB(t_2)] = \begin{cases} 0, & t_1 \neq t_2 \\ dt, & t_1 = t_2 = t \end{cases}. \quad (6)$$

Ito's stochastic differential rule is as follows. Let the p-th moments be $\boldsymbol{M}$,

$$M(X) = X_1^{k1} X_2^{k2}, \quad p = k_1 + k_2\,, \quad (7)$$

$$\frac{d}{dt}E[M] = E\left[X_2\frac{\partial M}{\partial X_1} + \left(-2\alpha X_2 + \pi K_{\eta\eta}X_2 - \Omega^2 X_1\right)\frac{\partial M}{\partial X_2} + \pi\left(K_{\gamma\gamma}X_1^2 + K_{\eta\eta}X_2^2 + K_{\xi\xi}\right)\frac{\partial^2 M}{\partial X_2^2}\right] \quad (8)$$

Steady state solution of the second order moments is

$$m_{20} = \frac{\pi K_{\xi\xi}}{2\Omega^2\left(\alpha - \pi K_{\eta\eta}\right) - \pi K_{\gamma\gamma}}, \quad m_{02} = \Omega^2 m_{20}, \quad m_{11} = 0 \quad (9)$$

By using a similar procedure, the fourth order steady state moments are obtained as follows:

$$\begin{aligned} m_{40} &= \frac{3\pi K_{\xi\xi}m_{20}}{\Delta m}\left[2\Omega^2 + 3A_{23}\left(\alpha - 2\pi K_{\eta\eta}\right)\right], \\ m_{31} &= 0, \quad m_{22} = \frac{\Omega^2}{3}m_{40}, \\ m_{13} &= \frac{2}{3}\left(\alpha - \pi K_{\eta\eta}\right)\Omega^2 m_{40} - \pi K_{\gamma\gamma}m_{40} - \pi K_{\xi\xi}m_{20}, \\ m_{04} &= A_{23}\left[2\left(\alpha - \pi K_{\eta\eta}\right)\Omega^2 - 3\pi K_{\gamma\gamma}\right]m_{40} \\ &\quad + \Omega^4 m_{40} - 3\pi A_{23}K_{\xi\xi}m_{20}, \end{aligned} \quad (10)$$

Where $\Delta m = A_{21} + A_{22} \times A_{23}$,
$A_{21} = \Omega^4(4\alpha - 7\pi K_{\eta\eta}) - 3\pi K_{\gamma\gamma}\Omega^2$,
$A_{22} = (2\alpha - 3\pi K_{\eta\eta})(\alpha - 2\pi K_{\eta\eta})$,
and $A_{23} = 6(\alpha - \pi K_{\eta\eta})\Omega^2 - 9\pi K_{\gamma\gamma}$.

Equivalent linearization coefficients in a quasi-linear system are selected to minimize the expected square of errors.

$$E[\{(2\alpha_1 - 2\alpha)Y + (\Omega_1^2 - \Omega^2)Y + \lambda\dot{Y}^3 + \delta Y^3\}^2] = \min. \quad (11)$$

The equivalent linearization coefficients can be obtained by partially differentiating Eq. (11) with respect to them and calculating expectation values.

$$\alpha = \alpha_1 + \frac{\lambda m_{04} + \delta m_{31}}{2m_{02}}, \quad \Omega^2 = \Omega_1^2 + \frac{\delta m_{40}}{m_{20}} \quad (12)$$

An energy method is presented as an alternative. Equating the probabilistic average of stiffness energy and damping energy of the original nonlinear system (2) with those of quasi-linear system (3),

$$E\left[\alpha \dot{Y}^2\right] = E\left[\alpha_1 \dot{Y}^2 + \frac{1}{4}\lambda \dot{Y}^4\right],$$
$$E\left[\frac{1}{2}\Omega^2 Y^2\right] = E\left[\frac{1}{2}\Omega_1^2 + \frac{1}{4}\delta Y^4\right] \quad (13)$$

The result is

$$\alpha = \alpha_1 + \frac{\lambda m_{04} + \delta m_{31}}{4m_{02}}, \quad \Omega^2 = \Omega_1^2 + \frac{\delta m_{40}}{2m_{20}} \quad (14)$$

Linearizing coefficients can be obtained by solving iteratively Eqs. (9), (10) and (12) or Eqs. (9), (10) and (14). Spectral density functions can be calculated from the real parts of the solution of equations that are formulated from the Fourier transforms of correlation functions and their differentials. Steady state moments (9) and (10) and spectral densities are the approximate solutions of the original nonlinear system (2) [5, 10].

2.2 Two DOF system

A uniform elastic beam with narrow rectangular section is considered. It is simply supported at both ends, v is the displacement in the y direction, u in the x direction, ϕ is the angle of twist, and receives random bending moments $M(t)$ at both ends. Let EI_x =bending stiffness in x axis, EI_y = bending stiffness in y axis, GJ =twisting stiffness, m =beam mass per unit length, ρ =polar radius of gyration of beam section, and α, β, γ are viscous damping coefficients. Governing equation of beam motion is as Eq. (15) [11].

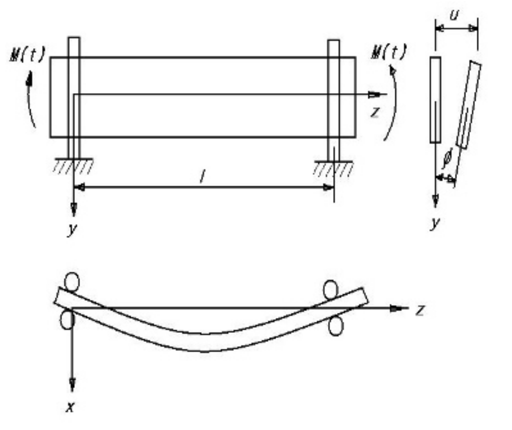

Fig. 1. A beam with narrow rectangular cross section.

$$EI_x \frac{\partial^4}{\partial z^4} v + m\ddot{v} + \alpha \dot{v} = 0$$
$$EI_y \frac{\partial^4}{\partial z^4} u + M(t)\frac{\partial^2}{\partial z^2}\phi + m\ddot{u} + \beta \dot{u} = 0 \quad (15)$$
$$M(t)\frac{\partial^2}{\partial z^2} u - GJ\frac{\partial^2}{\partial z^2}\phi + m\rho^2 \ddot{\phi} + \gamma \dot{\phi} = 0.$$

The u and ϕ motions are coupled, and uncoupled with v motion. For this beam, u and ϕ motions have practical concerns as it is stiff for bending displacement in x axis. Considering basic mode, introducing non-dimensional parameters, and assuming that damping forces due to lateral displacements are nonlinear with a cubic term of velocity and disturbance is applied to lateral displacement,

$$\ddot{Q}_1 + 2\alpha \dot{Q}_1 + \lambda \dot{Q}_1^3 + \omega_1^2 Q_1 = \omega_1 \omega_2 Q_2 \eta(t) + \xi(t),$$
$$\ddot{Q}_2 + 2\zeta_2 \omega_2 \dot{Q}_2 + \omega_2^2 Q_2 = \omega_1 \omega_2 Q_1 \eta(t) \quad (16)$$

where $\eta(t)$ and $\xi(t)$ are Gaussian white noises.

If a nonlinear system can be substituted with a linear system, responses of a nonlinear system can be predicted approximately by applying all techniques for linear systems [3]. By analyzing the nonlinear excitation term as it is given, we can maintain some accuracy of analysis.

Applying quasi-linearization technique to system (16),

$$\ddot{Q}_1 + 2\zeta_1 \omega_1 \dot{Q}_1 + \omega_1^2 Q_1 = \omega_1 \omega_2 Q_2 \eta(t) + \xi(t),$$
$$\ddot{Q}_2 + 2\zeta_2 \omega_2 \dot{Q}_2 + \omega_2^2 Q_2 = \omega_1 \omega_2 Q_1 \eta(t). \quad (17)$$

To get general forms, we take new variables with $X_1 = Q_1$, $X_2 = Q_2$, $X_3 = \dot{Q}_1$, $X_3 = \dot{Q}_1$, $X_4 = \dot{Q}_2$,

$$\frac{d}{dt} X_i = f_i(X) + \sum_j g_{ij}(X) W_j(t), \; i = 1, 2, 3, 4, \; j = 1, 2 \; . \quad (18)$$

From Ito's stochastic differential rule [5], differential equations are obtained as Eq. (19).

$$dX_i = \left(f_i + \pi \sum_l \sum_r \sum_s K_{ls} g_{rs} \frac{\partial g_{il}}{\partial X_r} \right) dt + \sum_j \sum_l \sum_s \sqrt{2\pi K_{ls} g_{il} g_{js}} \, dB_j(t) \quad (19)$$

where K_{ls} are cross-spectral densities of $W_l(t)$ and

$W_s(t)$.

Taking moments $M = m_{ijkl} = E[X_1^i X_2^j X_3^k X_4^l]$, we get an ensemble average equation of second order moments for the system (17) as Eq. (20).

$$\frac{d}{dt}E[M] = E\left[\sum_i \left(f_i + \pi\sum_l \sum_r \sum_s K_{ls} g_{rs}\frac{\partial g_{il}}{\partial X_r}\right)\frac{\partial M}{\partial X_i}\right] + \pi\sum_l \sum_s K_{ls} E\left[\sum_i \sum_j g_{il} g_{js}\frac{\partial^2 M}{\partial X_i \partial X_j}\right] \tag{20}$$

LHS of Eq. (20) is the time differential of n-th order statistical moment, $n=i+j+k+l$, RHS depends on the function type of f_i and g_{ij} .

Stationary moments are obtained from simultaneous equations with 10 variables.

$$m_{2000} = \frac{2\pi\zeta_2 K_{\xi\xi}}{\omega_1^3(4\zeta_1\zeta_2 - \pi\omega_1\omega_2 K_{\eta\eta}^2)},\quad m_{0020} = \omega_1^2 m_{2000},$$
$$m_{0200} = \frac{\pi\omega_1^2 K_{\eta\eta}}{2\zeta_2\omega_2} m_{2000},\quad \cdots \tag{21}$$

For fourth order moments, simultaneous equations with 35 variables are to be solved such as,

$$3m_{2020} - 2\zeta_1\omega_1 m_{3010} - \omega_1^2 m_{4000} = 0,$$
$$3m_{2011} - 2\zeta_2\omega_2 m_{3001} - \omega_2^2 m_{3100} = 0,$$
$$2\zeta_1\omega_1 m_{2020} - m_{1030} + \omega_1^2 m_{3010} - \pi K_{\eta\eta}\omega_1^2\omega_2^2 m_{2200} = \pi K_{\xi\xi} m_{2000},$$
$$3m_{0202} - 2\zeta_2\omega_2 m_{0301} - \omega_2 m_{0400} = 0. \tag{22}$$

Equivalent damping coefficients in quasi-linear system are selected to minimize the expectation value of the square of errors as follows:

$$E\{(2\alpha X_3 + \lambda X_3^3 - 2\zeta_1\omega_1 X_3)^2\} = \min. \tag{23}$$

The partial differential of Eq. (23) with respect to ς_1 is to be zero and

$$\zeta_1 = \frac{\alpha_1}{\omega_1} + \frac{\lambda m_{0040}}{2\omega_1 m_{0020}} \tag{24}$$

An alternative energy method is presented with the condition of the same probabilistic averages of damping energies of systems (16) and (11),

$$E\left[\zeta_1\omega_1 X_3^2\right] = E\left[\alpha X_3^2 + \frac{1}{4}\lambda X_3^4\right] \tag{25}$$

Arranging Eq. (25),

$$\zeta_1 = \frac{\alpha_1}{\omega_1} + \frac{\lambda m_{0040}}{4\omega_1 m_{0020}} \tag{26}$$

Linearized damping coefficients can be obtained from the iterative calculation of Eqs. (21), (22) and (24), or Eqs. (21), (22) and (26).

3. Monte carlo simulation

Monte Carlo simulation (MCS) is a general term which uses a series of random numbers [12-14]. The basic procedure of Monte Carlo simulation is the generation of random numbers, modeling the random excitations, numerical solution of system responses, and statistical management of the response output. In this study, the linear congruential generator is used for random number generation and based on the following iterative equation:

$$X_{n+1} = (A \times X_n + C) \bmod M \tag{27}$$

where X_{n+1} is a new random number, X_n an old one, A a multiplier, and C, M constants. The characteristics of the generated series are dependent on the constant numbers A, C, and M [13].

The Box-Muller method is applied to transform a uniform distribution to a normal distribution [12, 13]. The Runge-Kutta method is used to solve the system of equations to obtain the time history analysis of responses [15, 16]. Finally, the steady-state moments and spectral densities of responses are evaluated.

4. Results of numerical analysis

4.1 One DOF system

For the analysis of system (2), we analyzed in the case of $\alpha_1 = 0.4$, $\Omega_1 = 6.0$ and compared the results of MCS. Fig. 2 shows the expected values of the square of displacements $E[Y_1^2]$ for given δs in the case of $K_{\xi\xi} \approx 0$ and $K_{\gamma\gamma} \approx 0$. In the Figs. 2~13, "Lam" means λ and "Del" means δ . The results of MCS are located between those of the analytic method and those of the energy method. Fig. 3 shows $E[Y_1^2]$ of two λs for the case of $K_{\xi\xi} \approx 0$ and $K_{\gamma\gamma} \approx 0$. Fig. 4 shows the expected values of the square of displacements for the given λs and δs. The results of MCS are located between those of analytic method and those of energy method.

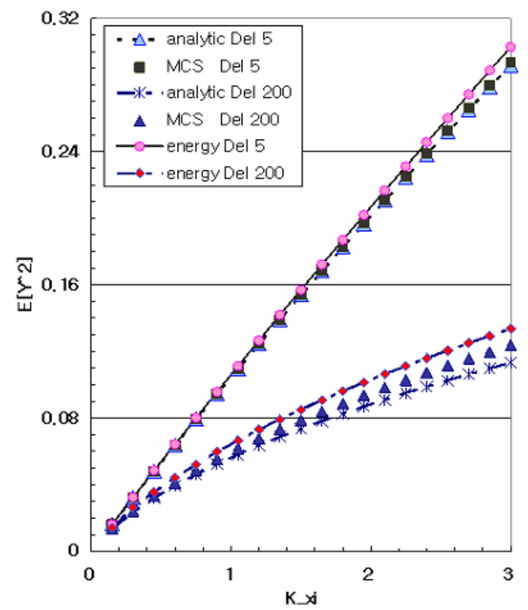

Fig. 2. Stationary mean square with δY^3 in the case of $\alpha_1 = 0.4, \Omega_1 = 6.0$, $K_{\xi\xi} \approx 0$ and $K_{\gamma\gamma} \approx 0$.

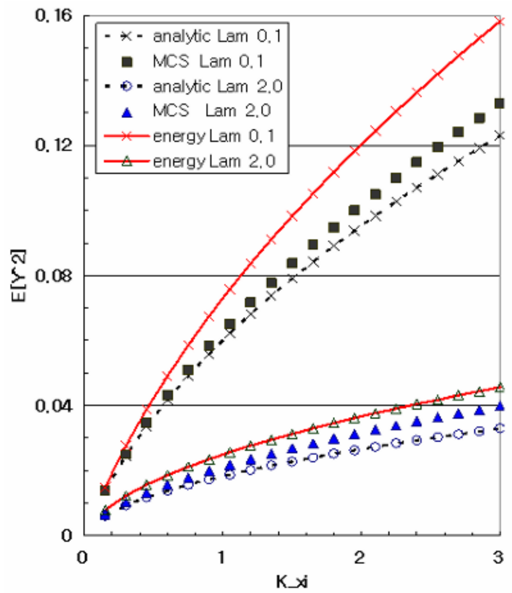

Fig. 3. Stationary mean square with $\lambda \dot{Y}^3$ in the case of $\alpha_1 = 0.4, \Omega_1 = 6.0$ $K_{\xi\xi} \approx 0$ and $K_{\gamma\gamma} \approx 0$.

Fig. 5 shows $E[Y_1^2]$ with λs and δs in the case of $\alpha_1 = 0.4, \Omega_1 = 6, K_{\gamma\gamma} = 0.5, K_{\eta\eta} = 0.05$. Fig. 6 shows $E[Y_1^2]$ with shows λs and δs in the case of $\alpha_1 = 0.8, \Omega_1 = 12, K_{\gamma\gamma} = 1.0, K_{\eta\eta} = 0.1$ and Fig. 7 shows spectral density $\phi_{11}(\omega)$ of the case in Fig. 6 where $K_{\xi\xi} = 1.5$. Results by MCS are located between those by the analytic method and those by the energy method in Figs. 5~7.

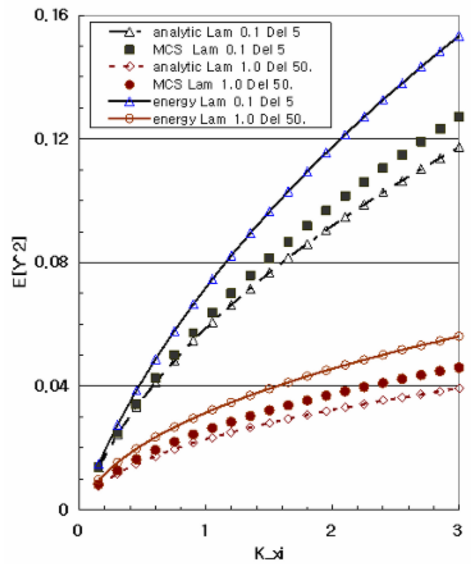

Fig. 4. Stationary mean square with $\lambda \dot{Y}^3$ and δY^3 in the case of $\alpha_1 = 0.4$, $\Omega_1 = 6.0$, $K_{\xi\xi} \approx 0$ and $K_{\gamma\gamma} \approx 0$.

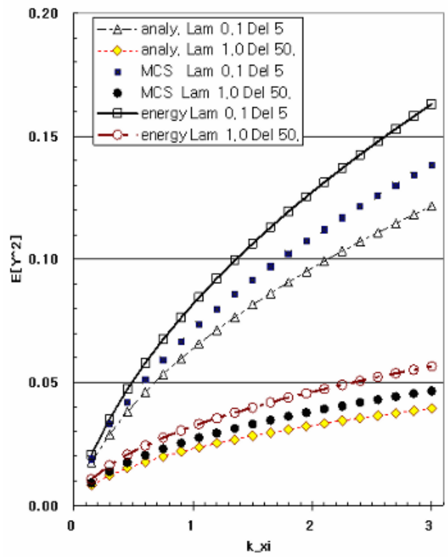

Fig. 5. Stationary mean square values of response in the case of $\alpha_1 = 0.4$, $\Omega_1 = 6.0$, $K_{\gamma\gamma} = 0.5, K_{\eta\eta} = 0.05$.

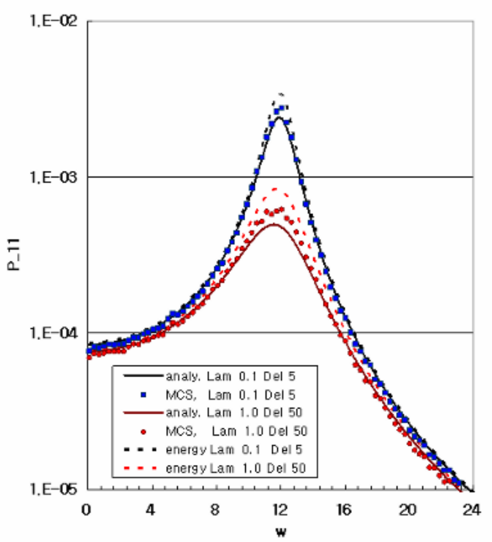

Fig. 6. Stationary mean square in the case of $\alpha_1 = 0.8$, $\Omega_1 = 12, K_{\gamma\gamma} = 1.0, K_{\eta\eta} = 0.1$.

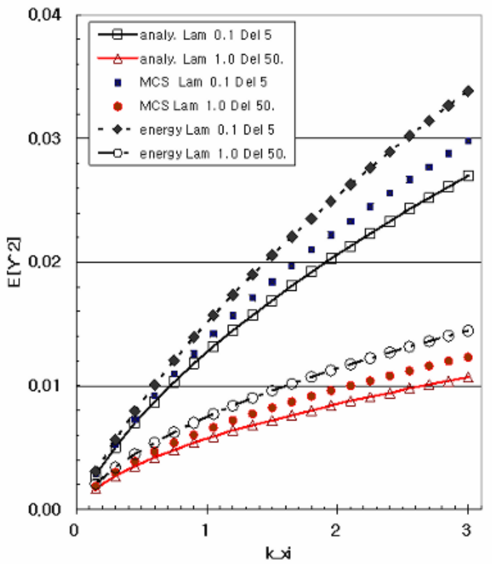

Fig. 7. Spectral density for $K_{\xi\xi}$ =1.5 in the case of $\alpha_1 = 0.8, \Omega_1 = 12, K_{\gamma\gamma} = 1.0, K_{\eta\eta} = 0.1$.

4.2 Two DOF system

The results of one DOF system gave the idea of forming a compromise of the above two method as Eq. (28).

$$\zeta_1 = \frac{\alpha_1}{\omega_1} + \frac{3\lambda m_{0040}}{8\omega_1 m_{0020}} \tag{28}$$

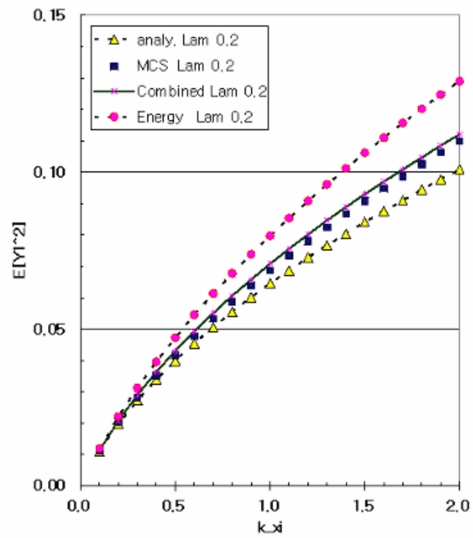

Fig. 8. Stationary mean square values of response with λ =0.2 and $K_{\eta\eta}$ =0.001 in the case of ω_1 =5, ω_2 =25, α =0.5, ξ_2 =0.1.

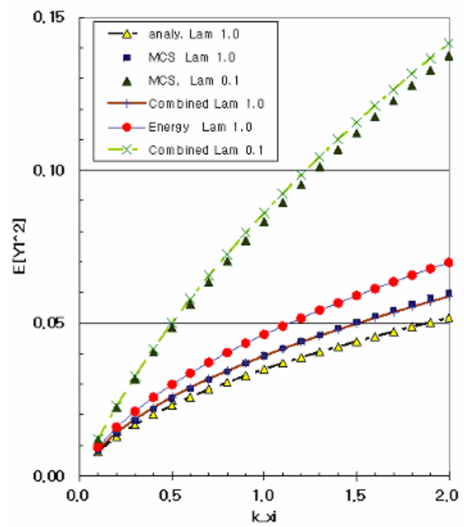

Fig. 9. Stationary mean square values of response with λ =0.1 or 1.0 and $K_{\eta\eta}$ =0.001 in the case of ω_1 =5, ω_2 =25, α =0.5, ξ_2 =0.1.

For the first numerical example of a two DOF system, we analyzed the case of ω_1 =5, ω_2 =25, α =0.5, ξ_2 =0.1 and compared with the results given by MCS. Figs. 8-9 show $E[Y_1^2]$ with $K_{\xi\xi}$ in the case of $K_{\eta\eta} = 0.001$. The value $\lambda = 0.2$, 0.1 and 1.0. Fig. 10 shows the case of $K_{\eta\eta} = 0.004$. The results of MCS moved towards those of the energy method from those of the analytical method as $K_{\eta\eta}$ increased.

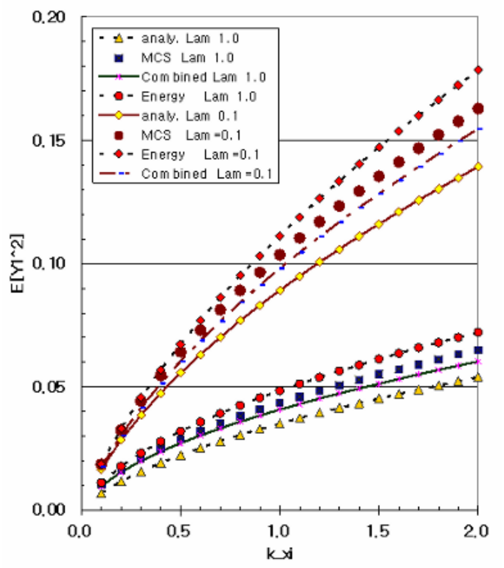

Fig. 10. Stationary mean square values of response with λ =0.1 or 1.0 and $K_{\eta\eta}$ =0.004 in the case of ω_1 =5, ω_2 =25, α =0.5, ξ_2 =0.1.

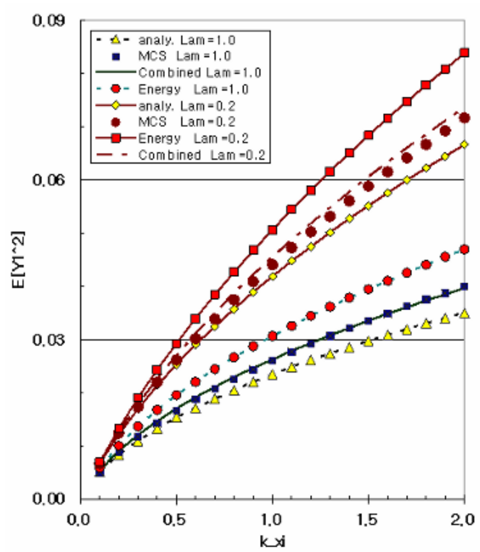

Fig. 11. Stationary mean square values of response with $K_{\eta\eta}$ =0.001, λ =0.2 or 1.0 in the case of $\omega_1=6$, $\omega_2=20$, $\alpha=0.6$, $\zeta_2=0.1$.

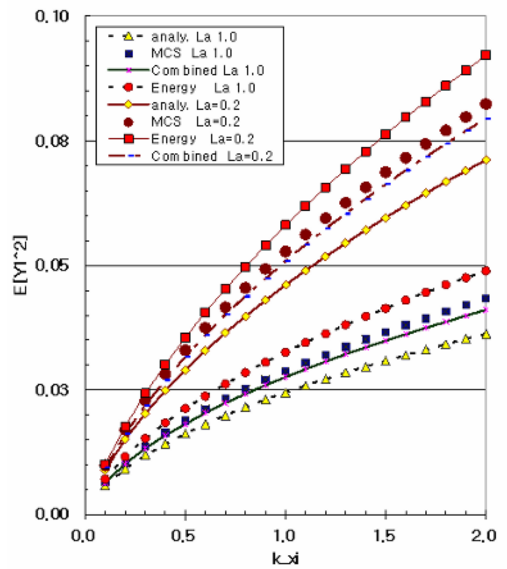

Fig. 12. Stationary mean square values of response with $K_{\eta\eta}$ =0.004, λ =0.2 or 1.0 in the case of $\omega_1=6$, $\omega_2=20$, $\alpha=0.6$, $\zeta_2=0.1$.

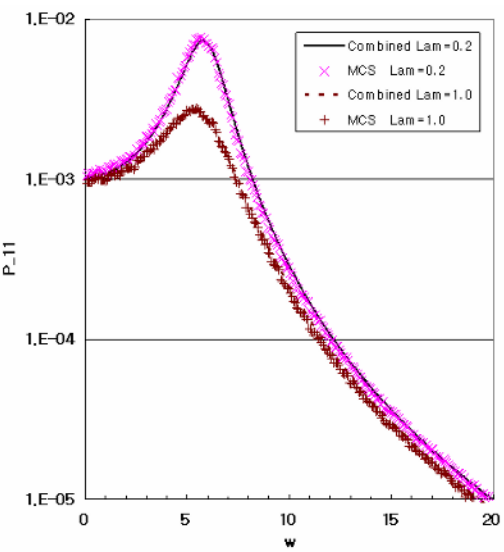

Fig. 13. Spectral density of response for the case of $K_{\xi\xi}$ =1.0, $K_{\eta\eta}$ =0.004, λ =0.2 or 1.0 in the case of $\omega_1=6$, $\omega_2=20$, $\alpha=0.6$, $\zeta_2=0.1$.

For the second numerical example, we analyzed the case of $\omega_1=6$, $\omega_2=20$, $\alpha=0.6$, $\zeta_2=0.1$, and compared with the results by MCS. Fig. 11 shows $E[Y_1^2]$ with $K_{\xi\xi}$ in the case of $K_{\eta\eta}=0.001$. Fig. 12 compares the three methods and MCS for the case of $K_{\eta\eta}=0.004$ and $\lambda=0.2$ or $\lambda=1.0$. As in Figs. 8~10**,** the results of compromise type are the nearest to those of MCS. Figs. 8-12 show that the analytical method gave lower limits, energy method upper limit, and the combination type the compromise values. The results by MCS show good agreement with the combination type. The spectral densities obtained by the combination type and MCS also show good agreement as shown in Fig. 13.

5. Concluding remarks

We used statistical methods to analyze nonlinear random vibration under parametric excitations. Quasi linear methods which linearize the non-linearity only of given systems were used. The analytical method which minimizes the expectation values of the square of errors showed the lower borderline. While the energy method which equates the expectation values of energies of the original systems and quasi-linearized systems showed the upper borderline. The combination type gave compromise results and showed good agreement with Monte Carlo simulation.

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